Bush 631-600: Quantitative Methods

Lecture 9 (11.01.2022): Probability vol. II

Rotem Dvir

The Bush school of Government and Public Policy

Texas A&M University

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What is today's plan?

- Calculating uncertainty: **probability**
- How probability is linked to our data.
- Random sample sums, means and their uncertainty.
- Large samples/data and their benefits for our analysis.
- Data management functions with tidyverse package.
- R work: table(), loops, simulations, plots.

We have findings!!!

- Data patters are systematic? Or noise?
- Our estimates $\rightarrow$ real relationship or random?

PROBABILITY:

- Set of tools to measure uncertainty in world (and our data).
- Method to formalize uncertainty or chance variation.
- Define odds for all possible outcomes.

Probability theory

Calculate probability of event:

$$P(A) = \frac{Elements(A)}{Elements(\Omega)}$$

Example: coin toss x 3

Get an least two heads?

Sample space (Ω): $\{$HHH,HHT,HTH,HTT,THH.THT,TTH,TTT$\}$.

Event A: $\{$HHH,HHT,HTH,THH$\}$.

Probability: $P(A) = \frac{4}{8} = 0.5$

Conditional probability

- We know event B occurred, what is the probability of event A?

$$P(A|B) = \frac{P(A\&B)}{P(B)}$$

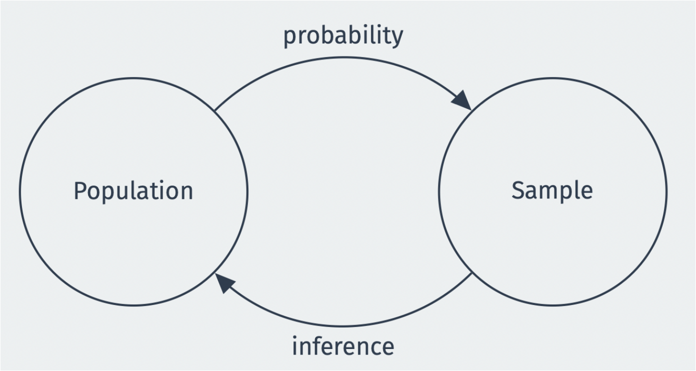
- Conditioning information matters:
  - Twins.
  - Monty hall problem (why switching is good..)

Independence

- Events are not related.
- Knowing the A occurred does not affect the probability of B occurring.
- Marginal probability of B (knowing A occurred) remains P(B).
- Formally:
  - $P(A\&B) = P(A) * P(B)$
  - $P(A|B) = P(A)$
  - $P(B|A) = P(B)$

Study probability

- Foundations for estimating quantities we care about.
- Making inferences from data to population

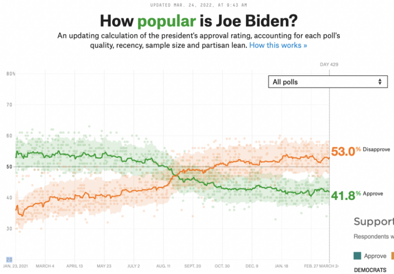


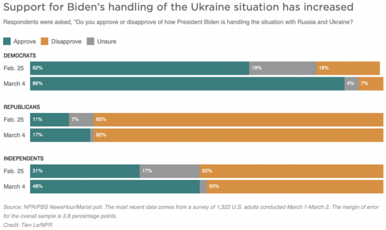
How did we get the data?

- Learn about the process that ‘generated our data’
- The role of uncertainty in this process

Approval data

How popular is president Biden?





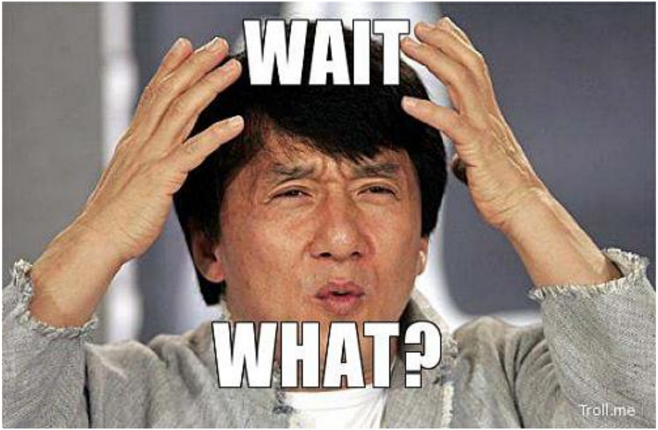
Random variables

- President's approval $\rightarrow$ public samples.
- Using probability to infer from sample to US population.
- The challenge: How to "draw" a Biden supporter?

$$\Downarrow$$

Use random variables to map outcomes to numbers

Random draws. . .



- Draw people???
- Random selection of values.

Random draws of. . . states

THE WORLD BANK
IBRD · IDA

Data > Data Catalog > World Development Indicators > Tables > 6.11

6.11 World Development Indicators: Aid dependency

☑ Show Metadata Links

	Net official development assistance				Aid dependency ratios		
	Total	per capita	Grants	Technical cooperation	Net official development assistance	Net official development assistance	Net official development assistance
	$ millions	$	$ millions	$ millions	% of GNI	% of gross capital formation	% of imports of goods, services and primary income
	2019	2019	2019	2019	2019	2019	2019
Afghanistan	4,284	113	3,915	300	21.9	..	57.8
Albania	28	10	150	103	0.2	..	0.4
Algeria	176	4	98	160	0.1	0.2	..
American Samoa	..	..	..	..	..	..	..
Andorra	..	..	..	..	..	..	..
Angola	50	2	158	47	0.1	..	0.2
Antigua and Barbuda	27	283	23	1	1.7	..	..
Argentina	18	0	49	43	0.0	0.0	0.0
Armenia	420	142	109	47	3.0	17.6	5.1

- Our objective: study regime type and extent of aid.
- Regimes: dictators, democracies, semi-democracies, etc.
- Draw regimes at-random and test causal mechanism.

Random draws, why?

Randomization:

- RCT: average all pre-treatment factors.
- RCT: strong causal explanation.

- Observational: reduce **selection bias**.
  - Allow expectations to be refuted.

We generate estimates, but with uncertainty

Numbers and Aggies example

Aggies in the NFL: position groups and conferences

```
skillposition <- subset(Ags, subset = (Group == "OF" | Group == "DF"))
head(skillposition)
```

```
## # A tibble: 6 x 5
##   Player             Team                  Position Group Conference
##   <chr>              <chr>                 <chr>    <chr> <chr>
## 1 Christian Kirk     Jacksonville Jaguars  WR       OF    NFC
## 2 Jake Matthews      Atlanta Falcons       OT       OF    NFC
## 3 Otaro Alaka        Baltimore Ravens      LB       DF    AFC
## 4 Justin Madubuike   Baltimore Ravens      DT       DF    AFC
## 5 Tyrel Dodson       Buffalo Bills         LB       DF    AFC
## 6 Germain Ifedi      Chicago Bears         OG       OF    NFC
```

Random variables and Aggs

```
## # A tibble: 2 x 2
##   Group     n
##   <chr> <int>
## 1 DF       12
## 2 OF       22
```

- Choose one at-random.
- Define **random variable**:
  - $X = 1$ if selected Aggie plays Offense, $X = 0$ otherwise.
- Why *random*?
- Before we draw an Aggie, uncertainty about the value of X.
- Linking to probability:
  - $P(X = 1) = \text{P(Draw Offense)} = \frac{22}{34} = 64.7\%$

Random variables

- Classified by construction and shape

BERNOULLI

- r.v. X follows a **bernoulli distribution** with probability p if:
  - X takes one of two values only (0,1).
- $P(X = 1) = p$
  - $P(X = 0) = 1 - p$
- Fits a binary indicator
- Describes **any** potential variable with a probability that $X = 1$.

Random variables

- Why?
  - The uncertainty of our estimates.
  - Figure the uncertainty of quantities as sample means or sums.
- Aggies data: drawing **two** players (with replacement):
  - $X_1 = 1$ if Aggie is Offense, $X_1 = 0$ otherwise.
  - $X_2 = 1$ if Aggie is Offense, $X_2 = 0$ otherwise.
- Define new r.v $\rightarrow S = X_1 + X_2$
- Data is the sum of all potential X_1, X_2.
- What are the values of S?

Random variables to probabilities

- Map S values to probabilities
- Always draw 2 Aggs.
- Sample space (Ω) = $\{$OF-OF; OF-DF; DF-OF; DF-DF$\}$.
- k $\rightarrow$ Values of S (0, 1, 2).
- $P(S = k)$?
- $P(S = k) = P(Ag_1 + Ag_2) = P(Ag_1) * P(Ag_2)$
- Why? Addition rule for mutually exclusive events.

Random variables to probabilities

```r
prob_off <- 22/34
prob_def <- 12/34

# Offense:Offense (OF-OF)
prob_off * prob_off
```

```
## [1] 0.4186851
```

```r
# Offense:Defense (OF-DF)
prob_off * prob_def
```

```
## [1] 0.2283737
```

```r
# Offense:Defense (DF-OF)
prob_def * prob_off
```

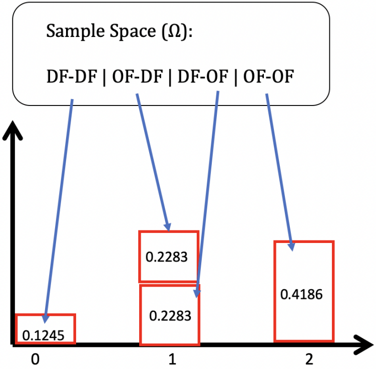
```
## [1] 0.2283737
```

```r
# Defense:Defense (DF-DF)
prob_def * prob_def
```

```
## [1] 0.1245675
```

Mapping draws to probabilities

Plotting probabilities of separate draws



Outcome	S	Probability
OF-OF	0	0.1245
OF-DF	1	0.2283
DF-OF	1	0.2283
OF-OF	2	0.4186

k	P(S = k)
0	0.1245
1	0.4567
2	0.4186

Binomial Distribution

- X is r.v. taking any value between 0 and n.
- Coin flips: number of heads with probability p in n independent flips.
- Aggs: S = number of OF when we draw **2 players** (n=2; P=0.4186).

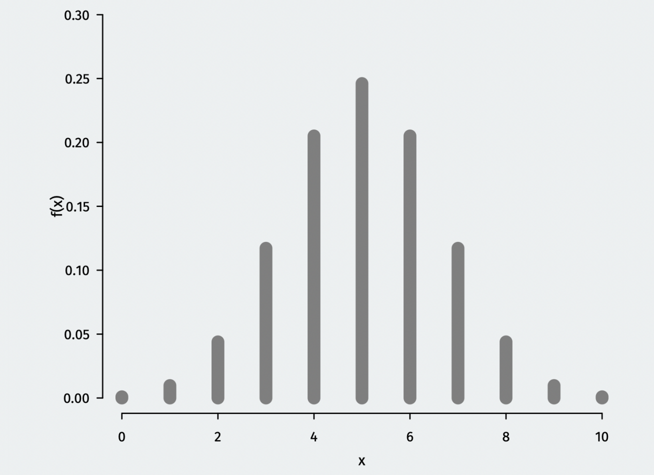
Probability Mass Function (PMF):

- Evaluates probability of *any possible value* of these random variables.

$$P(X = k) = \binom{n}{k} * p^k * (1 - p)^{n-k}$$
$$\binom{n}{k} = \frac{n!}{(k!(n-k)!)}$$

Binomial distribution

- X = number of heads in multiple coin flip trails
- $P = f(x) = 0.5$; $n = 10$



Binomial random variable

- Larger sample, more draws, same probability
- How many OF players?

```
# Possible number of Offensive players of 500
rbinom(n=3, size = 500, prob = 0.647)
```

```
## [1] 323 322 331
```

- Simulation

```
sims <- 10000
draws <- rbinom(sims, size = 500, prob = 0.647)
head(draws, n=8)
```

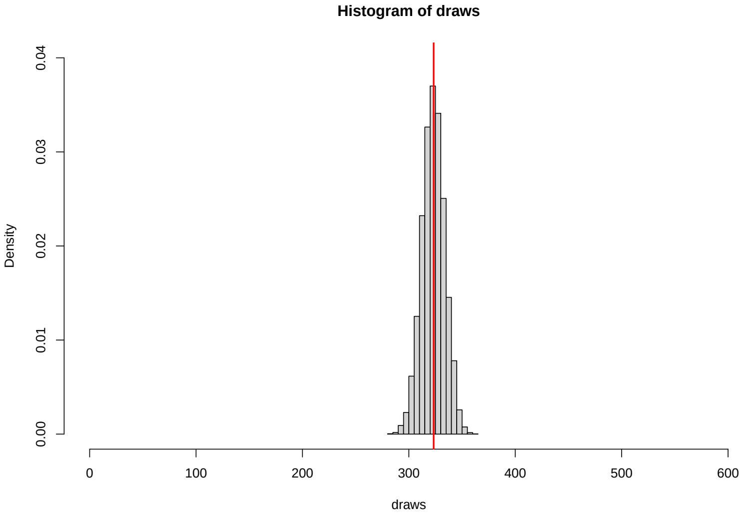
```
## [1] 327 322 324 332 351 315 313 330
```

```
mean(draws)
```

```
## [1] 323.4632
```

Plotting our sims

```
# Histogram of draws
hist(draws, freq = FALSE, xlim = c(0, 600), ylim = c(0, 0.04))
abline(v = 323.3, col = "red", lwd = 2)
```



Simulating Congress calls

- Lobbying firm: gender balance of calls to senators
- Total number of calls = 1000, random selection (with replacement)
- How many calls to women senators?

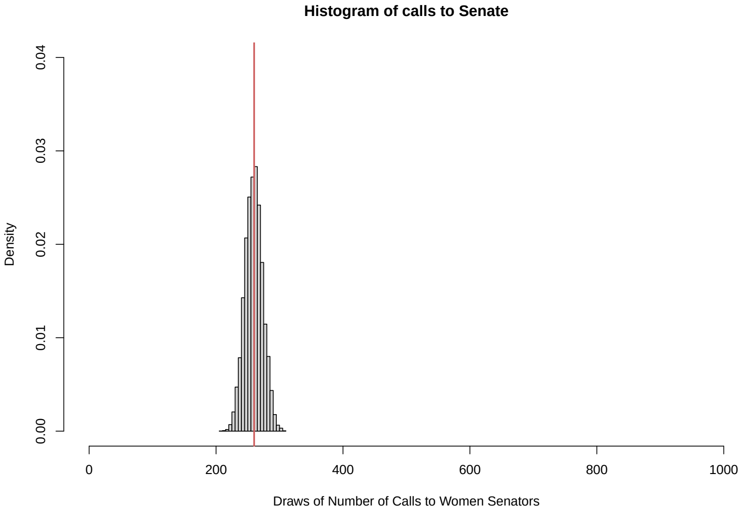
```
# Simulate calls (p=0.26)

sims2 <- 10000
draws2 <- rbinom(sims, size = 1000, prob = 0.26)
mean(draws2)
```

```
## [1] 259.9973
```

```
head(draws2, n=8)
```

```
## [1] 286 241 266 273 269 242 269 244
```

Plotting Senate calls simulation



Probability distributions

- Describe the uncertainty of random variables
- We learn of the population after analyzing the sample
- Example: draw random American adult.
  - r.v. X Bernoulli with probability p.
  - Define: $X = 1$ if TX resident, $X = 0$ otherwise.
- Finding p tell us the likelihood that a random American is from TX.

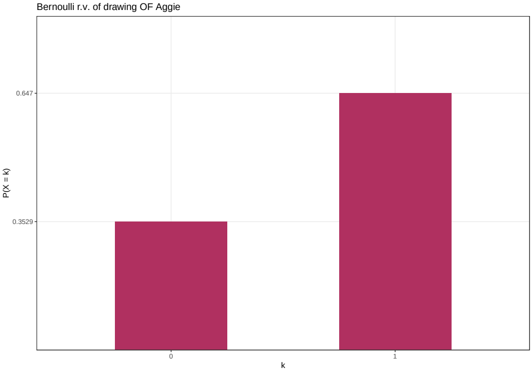
Probability distributions

- Multiple ways to represent the distribution.
- Type of r.v. $\rightarrow$ which distribution we face.
- Two general classes:
  - Discrete: X takes finite number of values (heads in n coin flips, battle deaths in civil wars).
  - Continuous: X takes any real value (GDP/cap, how long do you spend time on Tik-Tok?)

Discrete PMF

- Barplot to illustrate probabilities (share of each possible value)
- Bernoulli r.v.: using the Ags data (OF or DF?)

```
plot.dat <- data.frame(k = c("0", "1"), y = c("0.3529", "0.647"))
ggplot(plot.dat, aes(k,y)) +
  geom_bar(stat = "identity", width = 0.5, fill = "maroon") + ylab("P(X = k)")
  ggtitle("Bernoulli r.v. of drawing OF Aggie") + theme_bw()
```



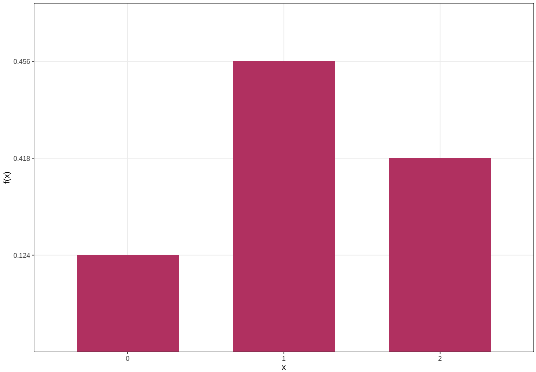
Binomial PMF

- Illustrate probabilities of 3 values (r.v. X)

```
dbinom(x = c(0,1,2), size = 2, prob = 22/34)
```

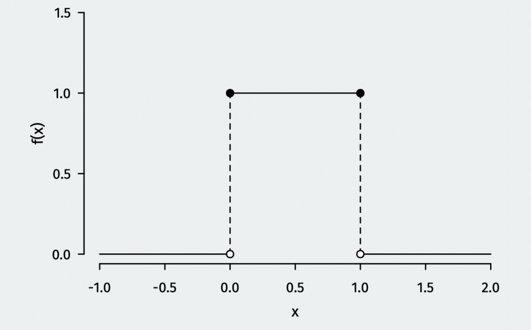
```
## [1] 0.1245675 0.4567474 0.4186851
```

```
plot.dat2 <- data.frame(x = c("0", "1", "2"), y = c("0.124", "0.456", "0.418"))
ggplot(plot.dat2, aes(x,y)) +
  geom_bar(stat = "identity", width = 0.65, fill = "maroon") + ylab("f(x)") +
  theme_bw()
```



Continuous random variables

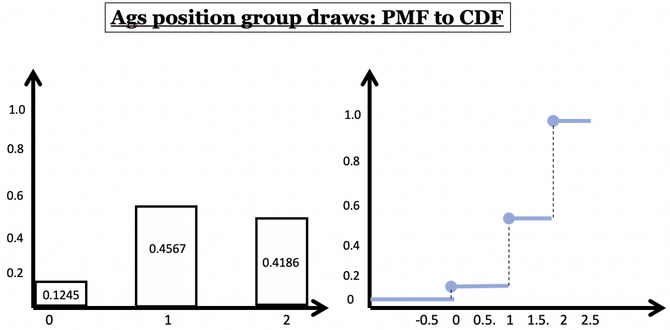
- **Probability density function (PDF)**.
- Describe probability 'around' a given point.
- An 'infinite' histogram $\rightarrow$ many bins (looks smooth).
- Probability of interval $=$ area under curve.



Random variable distributions

Cumulative distribution function (CDF).

- Common to discrete or continuous random variables.
- Describe the probability that some r.v. will be less or equal to some k.



Well, lets


Okay fine, you're right. Let's take a
break, let's cool off. Okay, let's get some
frozen yogurt or something.

Data management with tidyverse package

- Functions:
  - Organize data
  - Group values (mean, median)

```
# Insurgent groups (class tasks data)
head(DataManage)
```

```
## # A tibble: 6 x 27
##   torg_~1  torg  year group hbase hbccode gle_r~2 ucdpbd BIPOI~3 nukec~4 rev_f~5
##   <chr>   <dbl> <dbl> <chr> <chr>   <dbl>   <dbl>  <dbl>   <dbl>   <dbl>   <dbl>
## 1 4_1998      4  1998 Abu ~ Phil~     840   3443.     55       0       0       4
## 2 4_1999      4  1999 Abu ~ Phil~     840   3203.      0       0       0      11
## 3 4_2000      4  2000 Abu ~ Phil~     840   3202.    379       0       0      20
## 4 4_2001      4  2001 Abu ~ Phil~     840   3085.    333       0       0      55
## 5 4_2002      4  2002 Abu ~ Phil~     840   3026.    249       0       0      42
## 6 4_2003      4  2003 Abu ~ Phil~     840   3006.    129       0       0      28
## # ... with 16 more variables: rev_pm_fatalities <dbl>,
## #   rev_nonpm_fatalities <dbl>, reli <dbl>, left <dbl>, sepa <dbl>,
## #   fdstate <dbl>, crime <dbl>, terrcntrl <dbl>, stick <dbl>, age <dbl>,
## #   size_rec <dbl>, a_degree <dbl>, fh_ipolity2_inf <dbl>, gd_ptsa_inf <dbl>,
## #   attackculturalsite2 <dbl>, forBIPOICNEFFORT <dbl>, and abbreviated variable
## #   names 1: torg_year, 2: gle_rgdpc, 3: BIPOICNEFFORT, 4: nukecountry,
## #   5: rev_fatlties
## # i Use `colnames()` to see all variable names
```

Tidyverse the data

subsets → use filter()

```
# Subset groups operating in Iraq
sub.insurg <- DataManage %>%
  filter(hbase == "Iraq")

sub.insurg
```

```
## # A tibble: 65 x 27
##    torg_year  torg  year group      hbase hbccode gle_r~1 ucdpbd BIPOI~2 nukec~3
##    <chr>     <dbl> <dbl> <chr>      <chr>   <dbl>   <dbl>  <dbl>   <dbl>   <dbl>
##  1 51_2001      51  2001 Ansar Al-~ Iraq      645   4511.      0       0       0
##  2 51_2002      51  2002 Ansar Al-~ Iraq      645   3952.      0       1       0
##  3 51_2003      51  2003 Ansar Al-~ Iraq      645   2428.      0       0       0
##  4 51_2004      51  2004 Ansar Al-~ Iraq      645   3504.    143       1       0
##  5 51_2005      51  2005 Ansar Al-~ Iraq      645   3430.    302       0       0
##  6 51_2006      51  2006 Ansar Al-~ Iraq      645   3540.    160       0       0
##  7 51_2007      51  2007 Ansar Al-~ Iraq      645   3690.     25       0       0
##  8 51_2008      51  2008 Ansar Al-~ Iraq      645   3423.      0       0       0
##  9 51_2009      51  2009 Ansar Al-~ Iraq      645   4186.      0       0       0
## 10 51_2010      51  2010 Ansar Al-~ Iraq      645   3946.      0       0       0
## # ... with 55 more rows, 17 more variables: rev_fatlties <dbl>,
## #   rev_pm_fatalities <dbl>, rev_nonpm_fatalities <dbl>, reli <dbl>,
## #   left <dbl>, sepa <dbl>, fdstate <dbl>, crime <dbl>, terrcntrl <dbl>,
## #   stick <dbl>, age <dbl>, size_rec <dbl>, a_degree <dbl>,
## #   fh_ipolity2_inf <dbl>, gd_ptsa_inf <dbl>, attackculturalsite2 <dbl>,
## #   forBIPOICNEFFORT <dbl>, and abbreviated variable names 1: gle_rgdpc,
## #   2: BIPOICNEFFORT, 3: nukecountry
## # i Use `print(n = ...)` to see more rows, and `colnames()` to see all variable names
```

Tidyverse the data

add variables $\rightarrow$ use mutate()

```
# Create new var
# Define name, use ifelse to define values
DataManage <- DataManage %>%
  mutate(yrs90 = ifelse(year < 2000, "90s Rock","You're too old"))

prop.table(table(NewProp. = DataManage$yrs90))
```

```
## NewProp.
##       90s Rock You're too old
##      0.1103896      0.8896104
```

Tidyverse the data

Organize:

- Show specific columns → use select()
- Order the variable values → use arrange() (add - for high to low)

```
# Organize column (variable) by value (numeric or text)
DataManage %>% select(year,hbase,rev_fatlties) %>% head(n=4)
```

```
## # A tibble: 4 x 3
##    year hbase       rev_fatlties
##   <dbl> <chr>              <dbl>
## 1  1998 Philippines            4
## 2  1999 Philippines           11
## 3  2000 Philippines           20
## 4  2001 Philippines           55
```

```
DataManage %>% select(year,hbase,rev_fatlties) %>%
  arrange(-rev_fatlties) %>% head(n=4)
```

```
## # A tibble: 4 x 3
##    year hbase       rev_fatlties
##   <dbl> <chr>              <dbl>
## 1  2001 Afghanistan         2996
## 2  2012 Afghanistan         2548
## 3  2012 Nigeria             1252
## 4  2009 Iraq                1061
```

Tidyverse the data

Group variables & summary values:

- Creates **new** reduced dataset
- Calculate group mean, median, max...

```
# New dataset with summary stats for selected variables per group
new.dat <- DataManage %>%
  group_by(group) %>%
  summarise(fatal.mean = mean(rev_fatlties, na.rm = T),
            fatal.med = median(rev_fatlties, na.rm = T),
            mx.battle = max(ucdpbd, na.rm = T),
            mn.battle = min(ucdpbd, na.rm = T))
new.dat
```

```
## # A tibble: 140 x 5
##    group                                     fatal~1 fatal~2 mx.ba~3 mn.ba~4
##    <chr>                                       <dbl>   <dbl>   <dbl>   <dbl>
##  1 Abu Sayyaf Group (ASG)                     29.5        20     379       0
##  2 Al-Aqsa Martyrs Brigade                    26.2        13     126       0
##  3 Al-Fatah                                    2.53        0      72       0
##  4 Al-Gama'at Al-Islamiyya (IG)                0.467       0      27       0
##  5 Al-Ittihaad Al-Islami (AIAI)                2.38        0      25       0
##  6 Al-Nusrah Front                           309         309     339     339
##  7 Al-Qa'ida                                 256.         34    1585       0
##  8 Al-Qa'ida in the Arabian Peninsula (AQAP) 390.        293    2321      63
##  9 Al-Qa'ida in the Lands of the Islamic Maghre~ 113.    100     586     225
## 10 Al-Shabaab                                238.        206    2620       0
## # ... with 130 more rows, and abbreviated variable names 1: fatal.mean,
## #   2: fatal.med, 3: mx.battle, 4: mn.battle
## # i Use `print(n = ...)` to see more rows
```

Using r.v. distributions

- How to use probability distributions?
  - Mean: center of our distribution.
  - Variance/Standard deviation: the 'spread' around the center.
- Mean & Variance $\rightarrow$ *Population parameters* (unknown).
- Use our sample (data) to learn about both parameters.

Means & Expectations

Calculate the average: $\{1,1,1,3,4,4,5,5\}$

1. Common: sum all objects & divide by number of objects.

$$\frac{1+1+1+3+4+4+5+5}{8} = 3$$

2. Frequency weights: multiply each value by its frequency in the sample.

$$1 * \frac{3}{8} + 3 * \frac{1}{8} + 4 * \frac{2}{8} + 5 * \frac{2}{8} = 3$$

- Use the frequency weights approach to create the mean of r.v.s.

Expectation

- Expectation ($E[X]$) for the mean of r.v. X.

$$E[X] = \sum_{j=1}^{k} * x_j * P(X = x_j)$$

- The weighted average of the values of the r.v weighted by the probability of each value.

Expectation

- What is $E[X]$?
- Let X be the age for randomly selected individual.
- $E[X] \rightarrow$ average age in the *population*.
- $E[X]$: the link of the sample and population means.
- $E[X]$ properties:
  - $E[a] = a$ (constant).
  - $E[aX] = a * E[X]$ (scale for mean).
  - $E[aX + bY] = a * E[X] + b * E[Y]$ (mean of two values).

Variance

- The ‘spread’ of the distribution.

$$V[X] = E[(X - E[X])^2]$$

- Weighted avg. of squared distance if each observation from mean.
- Larger deviations $\rightarrow$ larger variance.
- If X be the age for randomly selected individual.
- $V[X] \rightarrow$ spread of ages in *population*.

Variance

- $SD(X) = \sqrt{V[X]}$: allows to make comparison in data.
- $V[X]$ properties:
  - $V[c] = 0$ (constant).
  - $V[aX + c] = a^2 * V[X]$ (scale distribution).
  - $V[X + Y] \neq V[X] + V[Y]$ (unless X & Y are independent).

Sums, means and random variables

- Let X_1 and X_2 be two r.v.s
- Then, $X_1 + X_2$ is also r.v.
- Mean: $E[X_1 + X_2]$; Variance: $V[X_1 + X_2]$

- We 'draw' two global leaders and assign X_1, X_2 as their ages.
- **Sample mean** $\rightarrow$ also a r.v.

$$\bar{X} = \frac{X_1 + X_2}{2}$$

- Uncertainty due to possibility of 'drawing' other leaders.

Global leaders data

- Data: personal characteristics of leaders (Horowitz 2015)

```
head(age.lead, n=9)
```

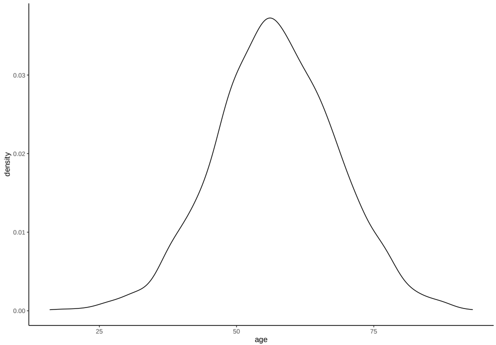
```
## # A tibble: 9 x 4
##   idacr  year leader           age
##   <chr> <dbl> <chr>          <dbl>
## 1 USA    1877 Grant             55
## 2 USA    1881 Hayes             59
## 3 USA    1881 Garfield          50
## 4 USA    1885 C. Arthur         56
## 5 USA    1889 Cleveland         52
## 6 USA    1893 Harrison          60
## 7 USA    1897 Cleveland         60
## 8 USA    1901 McKinley          58
## 9 USA    1909 Roosevelt, T.     51
```

Full sample means

```
# mean of sample
mean(age.lead$age, na.rm = T)
```

```
## [1] 57.122
```

```
# Plot distribution of all leaders in data
ggplot(age.lead, aes(x=age)) +
  geom_density() + theme_classic()
```



Distributions of sums & means

- 'Draw' two leaders, calculate sum and mean of age.

Drawing leaders at-random

	X_1	X_2	$X_1 + X_2$	Mean X
Draw 1	51 (Teddy R.)	69 (H.W.Bush)	120	60
Draw 2	55 (Rubio-MEX)	42 (Pardo – ECU)	97	48.5
Draw 3	69 (Chirac-FRN)	61 (Brandt-GFR)	130	65
Draw 4	38 (Delvina-ALB)	39 (Doe-LBR)	78	38.5
…	…	…	…	…

Distribution of sum

Distribution of mean

Independent and identical r.v.s

- $X_1 \ldots X_n$ are iid r.v.s.
- Random sample of n respondents on a survey question.
- **Identically distributed**: distribution of X_i is same for all i
  - $E(X_1) = E(X_2) = \ldots = E(X_n) = \mu$
  - $V(X_1) = V(X_2) = \ldots = V(X_n) = \sigma^2$
- Key insights of iid properties:
  - Sample mean = population mean (on average).
  - Variance $\leftarrow$ population variance and sample size.
  - SD of sample $\rightarrow$ *standard error*

$$SE = \sqrt{V[\bar{X}_n]} = \frac{\sigma}{\sqrt{n}}$$

Large samples: Global leaders

- We ‘draw’ two samples of global leaders
- Assign X_1, X_2 as their ages.
- Uncertainty of our data - leaders change each draw.

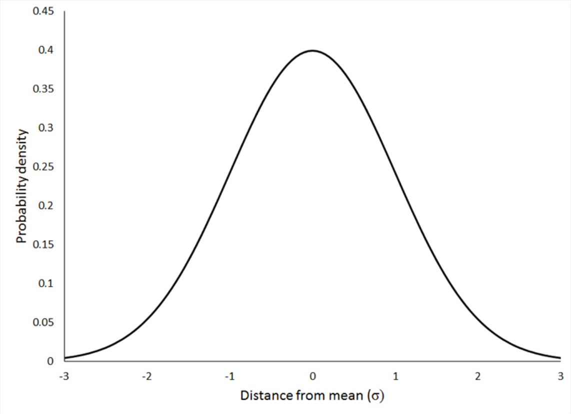
- What happens to our means when the sample size increases?

Large samples

Law of large numbers

- $X_1 ... X_n$ is iid with mean μ and variance σ^2.
- As n $\uparrow$, $\bar{x} \rightarrow \mu$.
- $P(\bar{x}) \rightarrow \mu$ increases as n get larger.
- Expectation: $E(\bar{X}) = E[X_i] = \mu$
- Think about the variance: $V(\bar{X}_n) = \frac{V[X]}{n}$

The Normal distribution



$$X \sim N(\mu, \sigma^2)$$

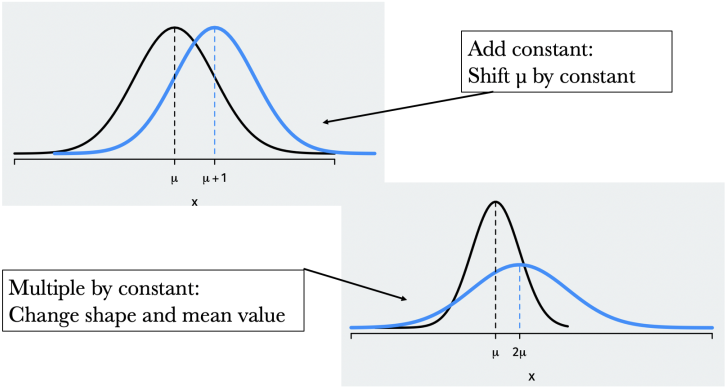
- Mean/expected value $= \mu$
- Variance $= \sigma^2$

The Normal distribution

- A "Bell-shaped" PDF
- Important properties:
  - Any r.v. is more likely to be in center than tails.
  - *Unimodal*: single peak, at the mean value.
  - Symmetric around the mean: equal probabilities.
  - Everywhere positive (tails 'stretch' to infinity).
- **Standard normal distribution**: mean $= 0$, SD $= 1$.
- Standard normal variable $\rightarrow$ *z-score*: $Z = \frac{X-\mu}{\sigma}$

The Normal distribution

- Transforming the normal distribution:



Central limit theorem

- Let X_i be r.v. which is iid and normally distributed.
- $\bar{X}$: also normally distributed in **large samples**.

Sample mean tend to be normally distributed as samples get large

- Extends the application of r.v. in large samples. How?
  - Value approaches μ and normally distributed.
  - Better approximation of population mean value.
  - Sample mean is normally distributed, regardless of the distribution of each X (r.v.).

Simulating larger sample (CLT)

- Draw at-random 1000 leaders from data.
- Calculate and save sample mean multiple times (use a loop)

```
sim.lead <- 1000
all.mn <- rep(NA, sim.lead)

for (i in 1:sim.lead){
  lead.draw <- sample_n(age.lead, 1000)
  all.mn[i] <- mean(lead.draw$age, na.rm = T)
}

head(all.mn)
```

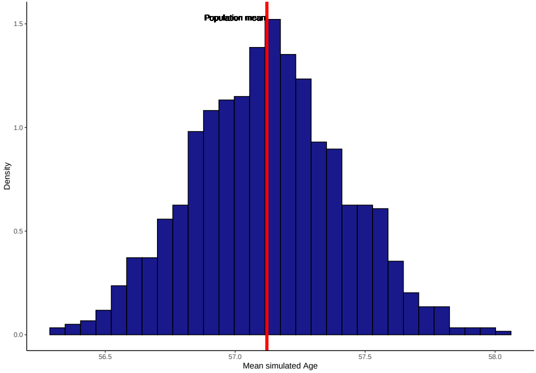
```
## [1] 57.24313 57.22983 56.98993 56.99695 57.13923 57.40793
```

```
mean(all.mn, na.rm = T)
```

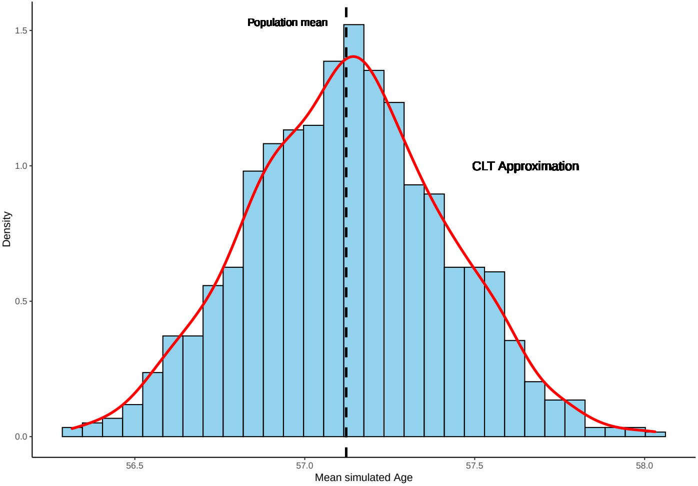
```
## [1] 57.12414
```

Plotting the simulated data

```
# Save vector in data frame and plot (add 'population' mean)
d <- data.frame(x = all.mn)
ggplot(d, aes(x)) +
  geom_histogram(aes(y = stat(density)),fill="navyblue", color="black", alpha=0.9) +
  xlab("Mean simulated Age") + ylab("Density") +
  geom_vline(xintercept = 57.122, color = "red", size = 2) +
  geom_text(aes(x = 57, y = 1.53, label = "Population mean")) +
  theme_classic()
```

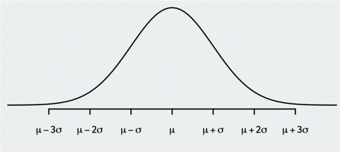


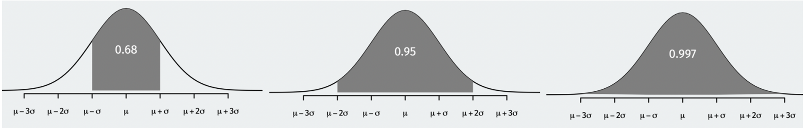
Plotting the simulated data



Empirical rule for normal distribution

If X $\sim$ N(μ, σ^2), then:





68% of dis. → 1 SD of mean

95% of dis. → 2 SD of mean

99% of dis. → 3 SD of mean

Empirical rule in R

```
# Values
pnorm(1) - pnorm(-1)
```

```
## [1] 0.6826895
```

```
pnorm(2) - pnorm(-2)
```

```
## [1] 0.9544997
```

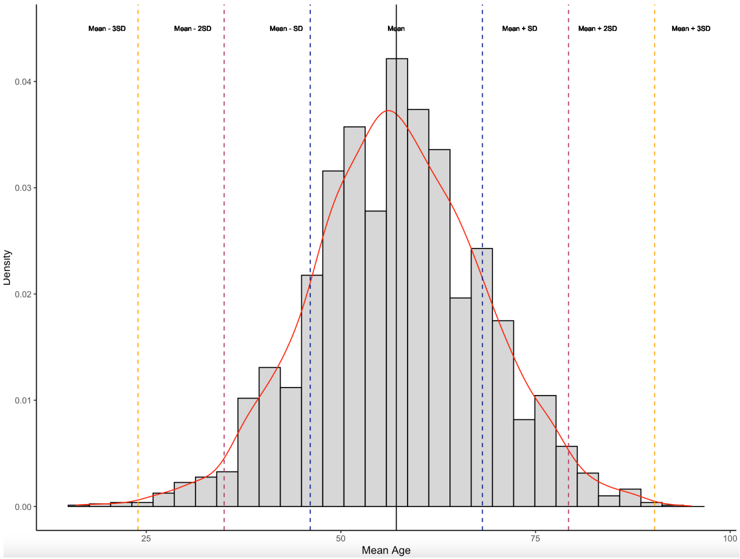
```
# Use the leader data
mu <- mean(Leader$age, na.rm = T)
sig <- sd(Leader$age, na.rm = T)

pnorm(mu+sig, mean = mu, sd = sig) - pnorm(mu-sig, mean = mu, sd = sig)
```

```
## [1] 0.6826895
```

```
pnorm(mu+2*sig, mean = mu, sd = sig) - pnorm(mu-2*sig, mean = mu, sd = sig)
```

```
## [1] 0.9544997
```

Leaders age: normal distribution “break-down”



Wrapping up week 9

Summary:

- Probability and uncertainty.
- Mapping probability of events to random variables.
- Linking r.v. to our data - random selection of values.
- Sums and means of random sample.
- Probability distributions (Bernoulli, Binomial, etc.).
- Large samples and their benefits.
- CLT / Law of large numbers.
- The normal distribution.

Research Proposal by Midnight!