

Bush 631-600: Quantitative Methods

Lecture 11 (11.15.2022): Uncertainty vol. II

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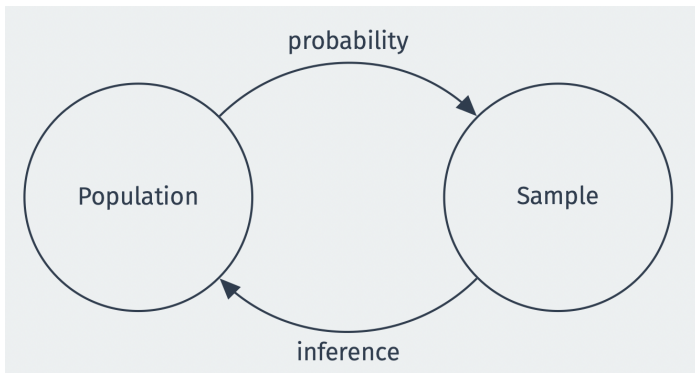
Fall 2022

What is today's plan?

- ▶ Calculating uncertainty: beyond measures of spread.
- ▶ Hypotheses tests
- ▶ When our results are 'significant'?
- ▶ Errors in testing.
- ▶ How to create efficient visual presentations?
- ▶ Power analysis.
- ▶ R work: `pnorm()`, `t.test()`, `power.prop.test()`, `power.t.test()`

Our data - our research interests

- ▶ Making inferences from data to population



Uncertainty

- ▶ Our proposed results:
 - ▶ Regime type matters for the extent of aid provision.
 - ▶ Financial incentives increase support for anti-plastic campaign.
- ▶ Are these effects real or just noise?

Estimation

- ▶ *Quantity of interest* in population.
- ▶ *Point estimation* → a 'best guess'.

$$\hat{\theta} = \bar{X}_n \rightarrow \text{population } \mu$$

- ▶ CLT and large samples → sample proportion = population.
- ▶ Experiments: diff-iin-means estimator.
- ▶ ATE in population.

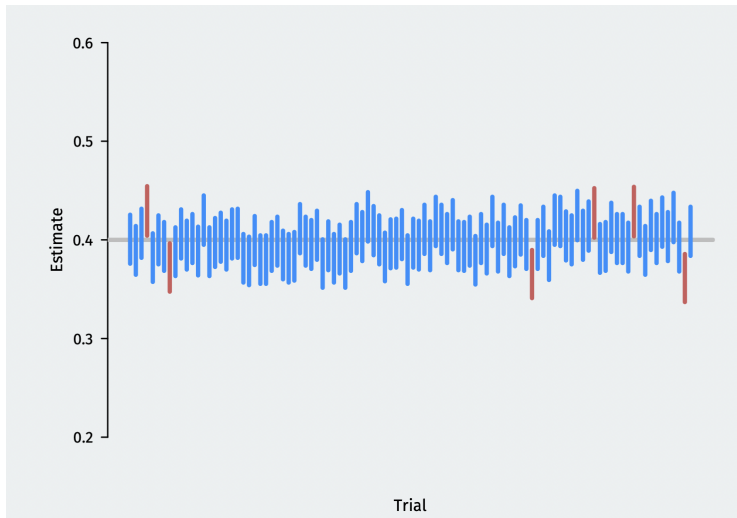
Uncertainty tools

- ▶ More ways to quantify uncertainty:
 - ▶ SD: variation of estimator ('spread' of distribution).
 - ▶ Only relevant for simulation.
 - ▶ Use SE for single sample.

- ▶ Confidence intervals:
 - ▶ Range of estimator true values.
 - ▶ The interpretation of CIs.

Simulate CIs

- ▶ How many overlap with 'true' support?



Testing our findings

- ▶ Describing uncertainty of results:
 - ▶ SD/SE.
 - ▶ Confidence intervals.

- ▶ Direct testing of results:
 - ▶ Hypothesis testing.
 - ▶ Significant effects.

The tea tasting experiment

- ▶ The scenarios:

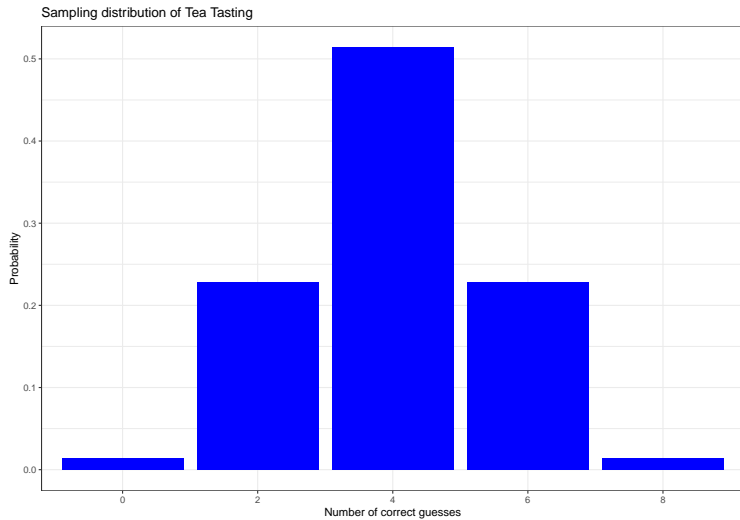
<i>Cups</i>	<i>Lady's guess</i>	<i>Actual order</i>	<i>Scenarios</i>			<i>...</i>
1	M	M	T	T	T	
2	T	T	T	T	M	
3	T	T	T	T	M	
4	M	M	T	M	M	
5	M	M	M	M	T	
6	T	T	M	M	T	
7	T	T	M	T	M	
8	M	M	M	M	T	
<i>Number of correct guesses</i>		8	4	6	2	<i>...</i>

Guessing?

- ▶ Only one way to choose **all 4 cups** correctly.
- ▶ But 70 ways to choose 4 among 8: ${}_8P_4 = \frac{8!}{4! \cdot (8-4)!}$
- ▶ Assume random choice: equal probability to each combination.
- ▶ Chances for guessing all 4 correctly: $\frac{1}{70} = 0.014 = 1.4\%$.

Does not seem like a guess...

Probabilities and guesses



Simulating tea tasting

- ▶ What are the odds of n correct cups?

```
# Simulations
sims <- 1000
guess <- c("M", "T", "T", "M", "M", "T", "T", "M")
correct <- rep(NA, sims)

for (i in 1:sims){
  cups <- sample(c(rep("T",4), rep("M",4)), replace = FALSE)
  correct[i] <- sum(guess == cups)
}

head(correct)

## [1] 4 2 6 6 4 6

prop.table(table(correct))

## correct
##      0      2      4      6      8
## 0.010 0.211 0.532 0.229 0.018
```

Statistical hypothesis testing

- ▶ **A thought experiment** - meaningful or random chance?
- ▶ Example:
 - ▶ You work in a research firm, study social equity.
 - ▶ Senior analyst claims 20% of Houston households are poor.
 - ▶ Data collection, sample of 950 households in Houston.
 - ▶ Mean in sample: 23% under poverty line.

Was the senior analyst wrong?

How certain are we that the sample is correct?

Statistical hypothesis testing

- ▶ Probabilistic *proof by contradiction*
- ▶ Assume the contrast to our expectations is not possible.



Proof by contradiction

- ▶ Houston research firm example
- ▶ Assume \rightarrow difference (sample and analyst) is zero.
- ▶ Incorrect? \rightarrow differences exist.
- ▶ Senior analyst may have been wrong.
- ▶ We can never **fully** reject a hypothesis (no 100% certainty).

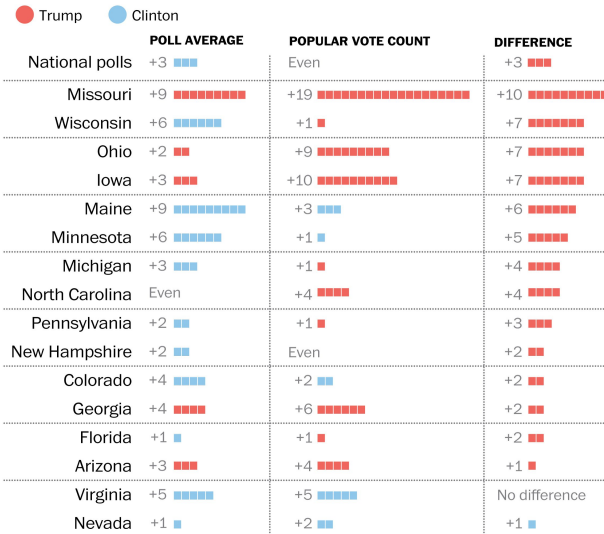
Procedure for hypothesis tests

- ▶ (1) Generate a **null hypothesis** (H_0).
- ▶ A statement we want to refute:
 - ▶ A devil's advocate position.
 - ▶ Red teams in strategic thinking/wargaming.
- ▶ (2) Define **alternative hypothesis** H_1/H_a .
- ▶ Statement of what we actually hope is true.
- ▶ Alternative hypothesis is *opposite* of null.
- ▶ (3) Show that differences are not due to chance.

Illustrating hypothesis testing

► 2016 elections polls

Trump consistently outperformed polls in key states



Illustrating hypothesis testing

- ▶ Now for 2020 Elections
- ▶ Finals polls: Trump support at 44%.
- ▶ Actual results: Trump support at 47.5%.
- ▶ Is 3% a real difference?

Our Q: true population support for Trump (p).

- ▶ Null hypothesis: $H_0 : p = 0.475$
- ▶ Alternative hypothesis: $H_1 : p \neq 0.475$
- ▶ Gather data: sample of about 1300 people.
- ▶ Mean proportion: $\bar{X} = 0.44$

Testing our null hypothesis

- ▶ If null is true, what the data distribution?
- ▶ $X_1 \dots X_n$ is Bernoulli with $p = 0.475$
- ▶ $X_j \dots X_n = 1$ if support Trump, 0 otherwise.
- ▶ Check with simulated data: share of votes in each sample.

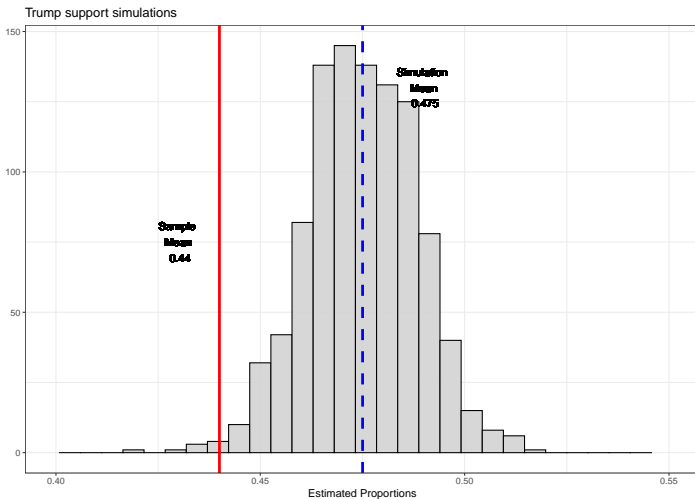
```
### Trump support simulation
t_vote <- rbinom(n = 1000, size = 1363, prob = 0.475)
t_share <- t_vote / 1363

head(t_share)
```

```
## [1] 0.4563463 0.4724872 0.4754219 0.4563463 0.4512106 0.4768892
```

Distribution of estimator

- ▶ Sample mean vs. simulated proportion of Trump support



Testing our null hypothesis

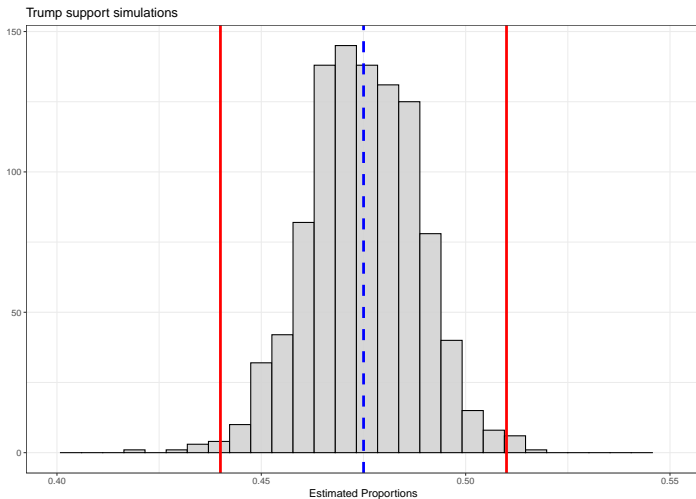
- ▶ Define *p-value*.
- ▶ Probability that observed data is as extreme as null.
- ▶ If Trump null is true. . .
- ▶ How often we get survey with big polling error?
 - ▶ Smaller *p-values* \rightsquigarrow stronger evidence against null.
 - ▶ **Not** probability that null is true!
- ▶ p-values are **two-sided**:
 - ▶ Error is 3.5% ($0.44 - 0.475 = -0.035$).
 - ▶ Check **both sides** of true mean ($p = 0.475$).

```
# Error (0.035) in both sides of mean (0.475)  
mean(t_share < 0.44) + mean(t_share > 0.51)
```

```
## [1] 0.013
```

Polling error odds

- ▶ 1% chance of polling error → not too likely



Hypothesis testing expanded

- ▶ **Directional** hypothesis
- ▶ Null is incorrect from 'one direction' only.
- ▶ Define hypotheses:
 - ▶ Null: $H_0 : p = 0.475$
 - ▶ **one-sided alternative**: $H_1 : p < 0.475$
 - ▶ $H_1 \rightarrow$ polls underestimate Trump support
- ▶ Define p-value as one-sided:
 - ▶ Probability that random sample underestimate Trump support as we see in our sample?
 - ▶ Smaller than two-sided p-value (only left side of distribution!)

```
# Error (0.035) is one-sided  
mean(t_share < 0.44)
```

```
## [1] 0.008
```

Final step in hypothesis tests

- ▶ Reject the null or not?
- ▶ Decide on *threshold* for rejection (test level α)
- ▶ **Decision rule:**
 - ▶ **Reject null** if p-value is *below* α
 - ▶ Otherwise **retain the null** or **fail to reject**.
- ▶ Common thresholds:
 - ▶ $p \geq 0.1$: “not statistically significant”.
 - ▶ $p < 0.05$: “statistically significant”.
 - ▶ $p < 0.01$: “highly significant”.

Decision rule

- ▶ Threshold → arbitrary (not a 'magic cut-off' points).
- ▶ $p = 0.051$ vs. $p = 0.049$??
- ▶ Function of data and sampling procedure.

Ronald Fisher. Journal of the Ministry of Agriculture of Great Britain (1926)

If one in twenty does not seem high enough odds, we may, if we prefer it, draw the line at one in fifty (the 2 per cent. point), or one in a hundred (the 1 per cent. point). Personally, the writer prefers to set a low standard of significance at the 5 per cent. point, and ignore entirely all results which fail to reach this level. A scientific fact should be regarded as experimentally established only if a properly designed experiment *rarely fails* to give this level of significance. The very high odds sometimes claimed for experimental results should usually be discounted, for inaccurate methods of estimating error have far more influence than has the particular standard of significance chosen.

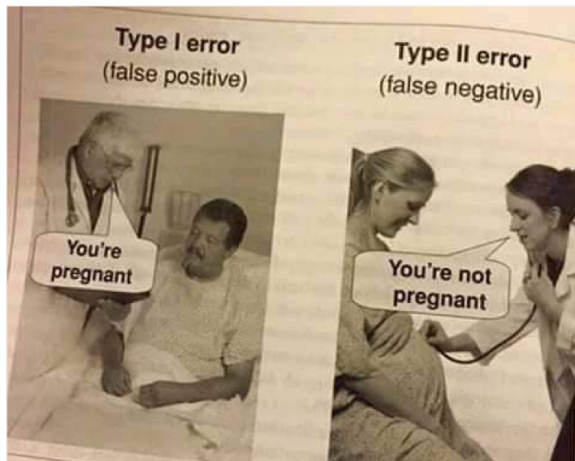
Test errors

- ▶ $p = 0.05 \rightarrow$ extreme data only happen in 5% of repeated samples (if null is true).
- ▶ \rightsquigarrow 5% of time we reject null that is true!
- ▶ Types of errors:

	H_0 True	H_0 False
Retain H_0	Awesome!	Type II error
Reject H_0	Type I error	Good stuff!

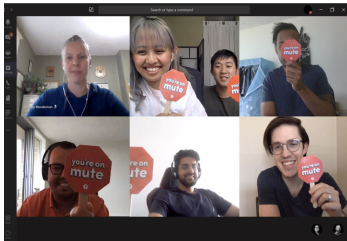
Test errors

- ▶ What does these errors mean?



Test errors

- ▶ Maybe more relevant??



Zoom to the rescue

Type I error

you think people can hear you, but you're actually on mute.

Type II error

you think your muted, but actually people can hear you.

Error types

- ▶ Trade-offs between error types:
 - ▶ What if we never reject the null?
 - ▶ $P(\text{type I}) = 0$.
 - ▶ Yet $\rightarrow P(\text{type II}) = 1$.
- ▶ Rejecting null: yes/no decision.
- ▶ p-value \rightarrow refute the null.
- ▶ Not a test of quantity of interest.

Statistical! Not scientific significance

Using visual tools

- ▶ Main objective:
 - ▶ Simple presentation of information.
 - ▶ Clear and straight forward.
 - ▶ “The eye test”: Humans favors color/pictures over numbers.

- ▶ Suggested guidelines:
 - ▶ How should my product look like?
 - ▶ Use graphics instead of text (if possible).
 - ▶ Show (me) the data: Annotate plots.
 - ▶ Start at zero!
 - ▶ Easy-to-read labels and captions.

Visual tools: Examples

- ▶ No, no, no, please NO!!!

Allergy Tablets

INDICATIONS: Provides effective, temporary relief of sneezing, watery and itchy eyes, and runny nose due to hay fever and other upper respiratory allergies.

DIRECTIONS: Adults and children 12 years and over—1 tablet every 4 to 6 hours, not to exceed 6 tablets in 24 hours or as directed by a physician. Children 6 to 11 years—one half the adult dose (break tablet in half) every 4 to 6 hours, not to exceed 3 whole tablets in 24 hours. For children under 6 years, consult a physician.

EACH TABLET CONTAINS: Chlorpheniramine Maleate 4 mg. **May also contain** (may differ from brand): D&C Yellow No. 10, Lactose, Magnesium Stearate, Microcrystalline Cellulose, Pregelatinized Starch.

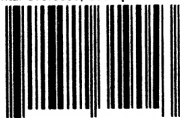
WARNINGS: May cause excitability especially in children. Do not take this product unless directed by a physician, if you have a breathing problem such as emphysema or chronic bronchitis, or if you have glaucoma or difficulty in urination due to enlargement of the prostate gland. May cause drowsiness; alcohol, sedatives and tranquilizers may increase the drowsiness effect. Avoid alcoholic beverages, and do not take this product if you are taking sedatives or tranquilizers without first consulting your physician. Use caution when driving a motor vehicle or operating machinery. As with any drug, if you are pregnant or nursing a baby, seek the advice of a health professional before using this product. Keep this and all drugs out of the reach of children. In case of accidental overdose, seek professional assistance or contact a Poison Control Center immediately.

Store at controlled room temperature 2°-30°C (36°-86°F).

- Use by expiration date printed on package.

Protect from excessive moisture.

For better identification keep tablets in carton until used.



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3 111111 11111 8

Made in U.S.A.

Visual tools: Examples

► Better...

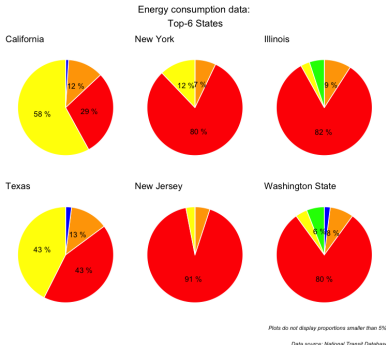
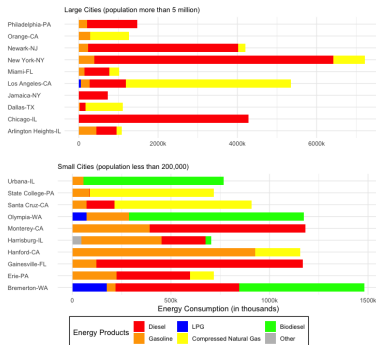
Drug Facts	
Active ingredient (in each tablet)	Purpose
Chlorpheniramine maleate 2 mg.....	Antihistamine
Uses temporarily relieves these symptoms due to hay fever or other upper respiratory allergies: ■ sneezing ■ runny nose ■ itchy, watery eyes ■ itchy throat	
Warnings	
Ask a doctor before use if you have	
■ glaucoma ■ a breathing problem such as emphysema or chronic bronchitis	
■ trouble urinating due to an enlarged prostate gland	
Ask a doctor or pharmacist before use if you are taking tranquilizers or sedatives	
When using this product	
■ drowsiness may occur ■ avoid alcoholic drinks	
■ alcohol, sedatives, and tranquilizers may increase drowsiness	
■ be careful when driving a motor vehicle or operating machinery	
■ excitability may occur, especially in children	
If pregnant or breast-feeding, ask a health professional before use.	
Keep out of reach of children. In case of overdose, get medical help or contact a Poison Control Center right away.	
Directions	
adults and children 12 years and over	take 2 tablets every 4 to 6 hours; not more than 12 tablets in 24 hours
children 6 years to under 12 years	take 1 tablet every 4 to 6 hours; not more than 6 tablets in 24 hours
children under 6 years	ask a doctor

Drug Facts (continued)	▲
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Visual tools: Examples

► Colors, captions to convey information

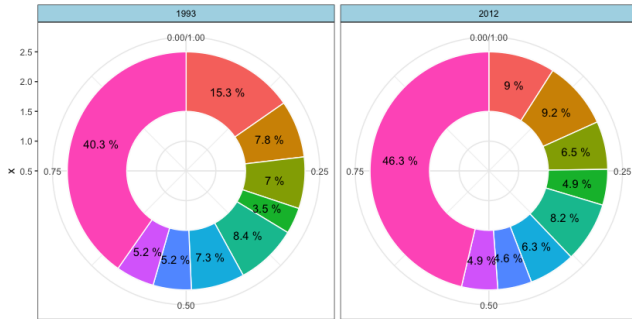
US Public Transportation System Energy Consumption (2019)



Visual tools: Examples

- Proportions with donuts. . .

US Gas Monthly consumption share: Top 8 states (June 1993; January 2012)



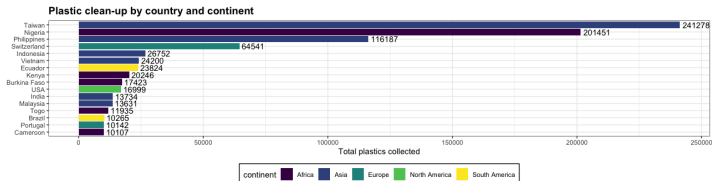
prop



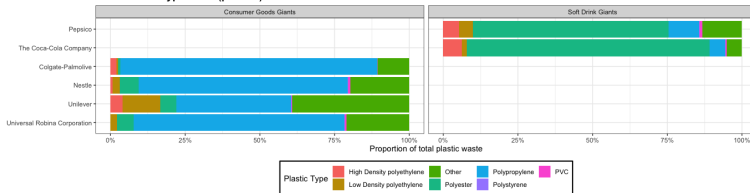
Visual tools: Examples

- Show the data: annotations. . .

'Break Free from Plastic' cleaning campaigns (2019-2020)



Plastic waste types: The (polluter) leader board



More info: breakfreefromplastic.org or [#breakfreefromplastic](https://twitter.com/breakfreefromplastic)

Visual tools: let's get technical

- ▶ The how?
 - ▶ Canva.
 - ▶ Piktochart.
 - ▶ Venngage.
 - ▶ Generate PDF.
 - ▶ Print at *Copy corner* (about \$10).

- ▶ More resources?
 - ▶ Useful tips → (VideoLink)
 - ▶ Cindy Raisor.

One sample tests

- ▶ Our sample mean - meaningful or random?
- ▶ Not just for binary r.v.s (Trump support).
- ▶ Steps for testing:
 1. Define null and alternative hyps ($H_0; H_1$).
 2. Select *test statistic* and level of test (α).
 3. Derive reference distribution of measure under null.
 4. Calculate p-values.
 5. Make a decision: reject/retain.

One sample tests

- ▶ **Test statistic:**

- ▶ Function of data.
- ▶ Used to assess both hypotheses.
- ▶ Most common → *z-score*

- ▶ The z-statistic:

$$Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}}$$

Or:

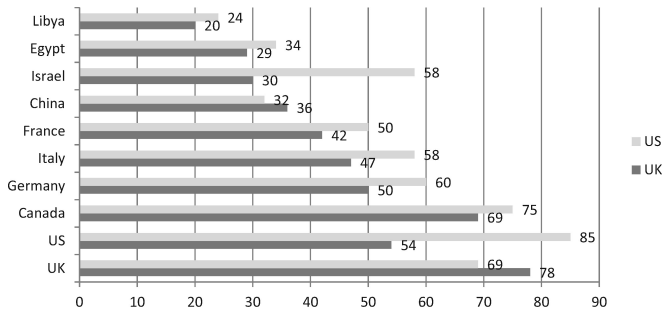
$$Z = \frac{\text{observed} - \text{null}}{SE}$$

- ▶ How many SEs away from the null guess is the sample mean?

Testing sample means

THERMOMETER SCORES

- ▶ Attitudes towards groups/concepts (0 ('cold') - 100 ('warm') scale).
- ▶ American attitudes to other nations (2017)



Americans attitudes: My Homecoming

- ▶ Focus on Israel.
- ▶ Score of 58 → in survey (Scotto and Reifler 2017).
- ▶ Recent surveys (Pew, 2019) → mean score of 55.
- ▶ Did attitudes shift? How 'real' are our survey results?
- ▶ Use hypothesis test!!

Testing public attitudes

- ▶ The set-up:

- ▶ $H_0 : \mu = 55$
- ▶ $H_1 : \mu \neq 55$

- ▶ Calculate the z-stat:

$$Z = \frac{\bar{X} - \mu}{\hat{\sigma} / \sqrt{n}} = \frac{58 - 55}{30 / \sqrt{800}} = 2.82$$

- ▶ Observed score is about 3 SEs from the null!!
- ▶ How unlikely is it??

Testing public attitudes data

$$Z = \frac{\bar{X} - \mu}{\hat{\sigma} / \sqrt{n}}$$

- ▶ Check the distribution of z under null.
- ▶ Not binomial, use CLT in large samples.
- ▶ $X \sim N(\mu, \sigma^2)$
- ▶ z is normal and follows the standard normal (mean 0, SD 1).

Testing public attitudes data in R

- ▶ What is the p-value of our test?

```
### Israel thermometer
# Define all values
n <- 800
x_bar <- 58
mu <- 55
sd <- 30
se <- (sd/sqrt(n))

# Calculate z-score (2.82)
z_ISR <- (x_bar - mu)/(sd/sqrt(n))

# What is the p-value?
pnorm(-z_ISR)

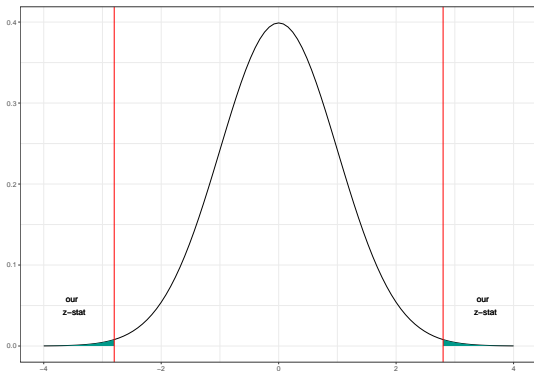
## [1] 0.002338867
```

Our survey plotted

- ▶ z-score (-2.8) and p-value (0.002)

```
# Create standard normal distribution data
p.norm <- data.frame(x = c(-4,4))

ggplot(p.norm, aes(x=x)) + stat_function(fun = dnorm) +
  geom_area(stat = "function", fun = dnorm, fill = "#00998a", xlim = c(-4,-2.8)) +
  geom_area(stat = "function", fun = dnorm, fill = "#00998a", xlim = c(2.8,4)) +
  geom_vline(xintercept = -2.8, color = "red") +
  geom_vline(xintercept = 2.8, color = "red") + xlab("") + ylab("") +
  geom_text(aes(x=-3.5, y=0.05, label = "our \n z-stat")) +
  geom_text(aes(x=3.5, y=0.05, label = "our \n z-stat")) + theme_bw()
```



Small samples problem

- ▶ z-score is useful for large samples under CLT.
- ▶ Small sample \rightarrow uncertainty about \bar{X} distribution.
- ▶ Assume X_i is normally distributed (not likely!)
- ▶ Find t-statistic instead:

$$T = \frac{\bar{X} - \mu}{\hat{SE}} \approx t_{n-1}$$

- ▶ t-distribution with $n-1$ degrees of freedom.
- ▶ Centered around zero.

Test with t-distribution

- ▶ t-test vs. z-test:
 - ▶ z → normal distribution (large samples, $n > 30$).
 - ▶ t → t-distribution (smaller samples, more conservative).
 - ▶ In t-distribution: larger p-values (less likely to reject null).
- ▶ One more testing option: construct CIs
- ▶ If $\mu = 55$ is not in CIs → **reject null!**

```
# Construct 95% and 99% CIs for x_bar = 58
```

```
c(x_bar - qnorm(0.995) * se, x_bar + qnorm(0.995) * se)
```

```
## [1] 55.26792 60.73208
```

```
c(x_bar - qnorm(0.975) * se, x_bar + qnorm(0.975) * se)
```

```
## [1] 55.92114 60.07886
```

Test for proportions

- ▶ Trump support numbers again: survey (0.44); actual (0.475).
- ▶ `prop.test()`: calculate p-value and 95% CIs.

```
# 600 supports in 1363 sample (x-bar)  
prop.test(600, n = 1363, p = 0.475, correct = FALSE)
```

```
##  
## 1-sample proportions test without continuity correction  
##  
## data: 600 out of 1363, null probability 0.475  
## X-squared = 6.6171, df = 1, p-value = 0.0101  
## alternative hypothesis: true p is not equal to 0.475  
## 95 percent confidence interval:  
## 0.4140563 0.4666907  
## sample estimates:  
##  
## p  
## 0.4402054
```

Two sample tests

- ▶ Goal: learn about population difference in means.
- ▶ Compare differences b-w multiple groups: same testing procedures.
- ▶ Define:
 - ▶ Null H₀: $\mu_T - \mu_C = 0$
 - ▶ Alt. H₁: $\mu_T - \mu_C \neq 0$
 - ▶ Test statistic: diff-in-means estimator.
 - ▶ z-score for *two sample z-test*.
- ▶ Are the differences in sample means just random chance?

Two sample test: generating data

- ▶ Generate experimental data (recipient regime and foreign aid)
- ▶ Focus on treatment 1 (regime type) & continuous DV (extent of aid)

```
head(exp.dat, n=8)
```

##	ID	trt1	trt2	dv_cor1	dv_cor2	cont_cor1	cont_cor2
## 1	0001	1	0	1	0	1523.100	1395.533
## 2	0002	1	1	1	1	1492.402	1466.578
## 3	0003	1	0	1	0	1500.165	1431.904
## 4	0004	1	0	1	0	1510.011	1406.666
## 5	0005	0	1	0	1	1515.649	1442.158
## 6	0006	0	0	0	0	1512.053	1430.640
## 7	0007	0	0	0	1	1474.265	1451.380
## 8	0008	1	1	1	0	1498.759	1443.719

Two sample test

- ▶ Define groups to compare in the data
- ▶ Run a **two sample t-test**

```
t.test(exp.dat$cont_cor1[exp.dat$trt1 == 0],
       exp.dat$cont_cor1[exp.dat$trt1 == 1])

##
## Welch Two Sample t-test
##
## data:  exp.dat$cont_cor1[exp.dat$trt1 == 0] and exp.dat$cont_cor1[exp.dat$trt1 == 1]
## t = -13.697, df = 993.53, p-value < 2.2e-16
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -23.59653 -17.68267
## sample estimates:
## mean of x mean of y
## 1489.333 1509.973
```

Two sample test

- ▶ Hawk/Dove leader and FP actions (Mattes and Weeks 2019)

```
# The data
head(hawksdata, n=3)

## # A tibble: 3 x 32
##   caseid hawk_t party_t rappr~1 succe~2 hawk   intl   trust voted16 polac~3
##   <dbl> <dbl+1> <dbl+1> <dbl+1> <dbl+1> <dbl+1> <dbl+1> <dbl+1> <dbl+1> <dbl+1>
## 1 3.29e8 2 [Dip~ 1 [Rep~ 1 [Red~ 1 [Pul~ 4 [Agr~ 4 [Agr~ 2 [The~ 4 [Yes~ 1 [sel~
## 2 3.29e8 1 [Mil~ 1 [Rep~ 1 [Red~ 1 [Pul~ 2 [Dis~ 4 [Agr~ 2 [The~ 4 [Yes~ 2 [not~
## 3 3.29e8 1 [Mil~ 2 [Dem~ 1 [Red~ 1 [Pul~ 2 [Dis~ 2 [Dis~ 2 [The~ 4 [Yes~ 1 [sel~
## # ... with 22 more variables: polact_2 <dbl+1>, polact_3 <dbl+1>,
## #   polact_4 <dbl+1>, hddv1 <dbl+1>, hdmed1_strat <dbl+1>,
## #   hdmed1_pacifist <dbl+1>, hdmed1_warmonger <dbl+1>, hddv2 <dbl+1>,
## #   hdmed2_strat <dbl+1>, hdmed2_pacifist <dbl+1>,
## #   hdmed2_warmonger <dbl+1>, birthyr <dbl>, gender <dbl+1>,
## #   educ <dbl+1>, pid3 <dbl+1>, pid7 <dbl+1>, ideo5 <dbl+1>,
## #   newsint <dbl+1>, pew_religimp <dbl+1>, approve_b <dbl>, ...
## # i Use `colnames()` to see all variable names

# Groups of support by Hawk/Dove treatment
table(hawksdata$hawk_t, hawksdata$hddv1)

##
##      1  2  3  4  5
## 1  59 132 148 187 74
## 2  73  83  83 217 143
```

Two sample test

- ▶ Does type (hawk/dove) makes a difference?
- ▶ Use **t-test**: compare support (1-5 scale) b-w groups

```
# t-Test
```

```
t.test(hawksdata$hddv1[hawksdata$hawk_t == 1],  
       hawksdata$hddv1[hawksdata$hawk_t == 2])
```

```
##
```

```
## Welch Two Sample t-test
```

```
##
```

```
## data: hawksdata$hddv1[hawksdata$hawk_t == 1] and hawksdata$hddv1[hawksdata$
```

```
## t = -4.3646, df = 1183, p-value = 1.385e-05
```

```
## alternative hypothesis: true difference in means is not equal to 0
```

```
## 95 percent confidence interval:
```

```
## -0.4577038 -0.1738210
```

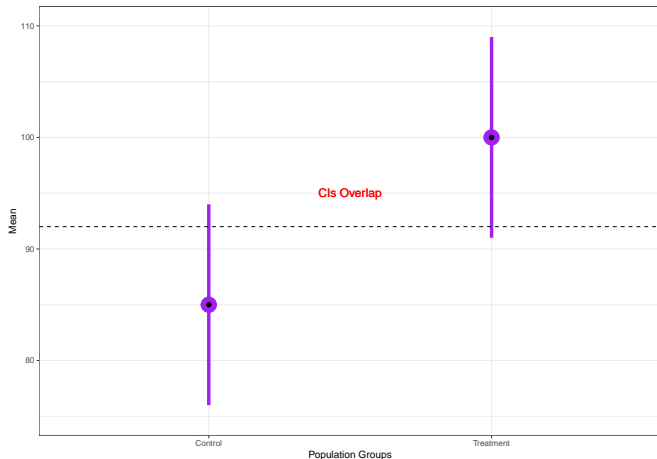
```
## sample estimates:
```

```
## mean of x mean of y
```

```
## 3.141667 3.457429
```

Testing failures

- ▶ We fail to reject null, but null is still untrue. Why?
- ▶ Sample is too small → 'inflates' SEs.



Power analysis

- ▶ **Power Analysis:**

- ▶ Power = $1 - P(\text{type II error})$.
- ▶ Type II error: *false negative*.
- ▶ Maximize test power!!!

- ▶ Calculate smallest sample size to identify differences.
- ▶ Like MOE discussion - initial step in survey design
- ▶ Minimum sample size to reject the null:
 - ▶ Surveys: similar proportion b-w groups (candidates).
 - ▶ Experiments: reject zero ATE.

Finding the power...

► Define:

1. Hypothetical sample size.
2. Probability of rejecting null (p-value, α).
3. Proportions of sample and 'population'.

► Calculate power of test (preferred power > 0.8).

```
# Define parameters and values
n <- 250
p_bar <- 0.47
p <- 0.5
alpha <- 0.05
cr_value <- qnorm(1 - alpha/2)
se_bar <- sqrt(p_bar * (1-p_bar) / n)
se_p <- sqrt(p * (1-p) / n)

# power
pnorm(p - cr_value * se_p, mean = p_bar, sd = se_bar) +
  pnorm(p + cr_value * se_p, mean = p_bar, sd = se_bar, lower.tail = F)
```

```
## [1] 0.1572895
```

Now for the power analysis

- ▶ Find sample size: `power.prop.test()` function.
- ▶ Define: Groups proportions (p_1, p_2); significance level (α)' power of test.

```
# What is minimum group size? (power = 0.9)  
power.prop.test(p1 = 0.5, p2 = 0.45, sig.level = 0.05, power = 0.9)
```

```
##  
##      Two-sample comparison of proportions power calculation  
##  
##              n = 2094.153  
##              p1 = 0.5  
##              p2 = 0.45  
##      sig.level = 0.05  
##              power = 0.9  
##      alternative = two.sided  
##  
## NOTE: n is number in *each* group
```


Power analysis

► Smaller proportions differences

```
# What is minimum group size? (power = 0.9)  
power.prop.test(p1 = 0.05, p2 = 0.1, sig.level = 0.05, power = 0.8)
```

```
##  
##      Two-sample comparison of proportions power calculation  
##  
##              n = 434.432  
##              p1 = 0.05  
##              p2 = 0.1  
##      sig.level = 0.05  
##              power = 0.8  
##      alternative = two.sided  
##  
## NOTE: n is number in *each* group
```

Power analysis

- ▶ Using `power.prop.test()` to find the test power

```
# Define sample size and groups proportions  
power.prop.test(n = 100, p1 = 0.05, p2 = 0.1, sig.level = 0.05)
```

```
##  
##      Two-sample comparison of proportions power calculation  
##  
##              n = 100  
##              p1 = 0.05  
##              p2 = 0.1  
##      sig.level = 0.05  
##              power = 0.2674798  
##      alternative = two.sided  
##  
## NOTE: n is number in each group
```

Power analysis

- ▶ Continuous variables: *t*-test version of power analysis.
- ▶ One sample: define mean (δ) and SD of r.v. in sample.
- ▶ Null: $H_0 : \bar{X} = 0$

```
# Minimum sample size  
power.t.test(delta = 0.3, sd = 1, type = "one.sample", power = 0.9)
```

```
##  
##      One-sample t test power calculation  
##  
##              n = 118.6865  
##            delta = 0.3  
##              sd = 1  
##      sig.level = 0.05  
##            power = 0.9  
## alternative = two.sided
```

Power analysis

- ▶ Two sample: define diff-in-means (delta) and SD.
- ▶ Null: $H_0 : \bar{X}_T - \bar{X}_C = 0$

```
# Minimum sample size  
power.t.test(delta = 0.3, sd = 1, type = "two.sample",  
             alternative = "one.sided", power = 0.85)
```

```
##  
##       Two-sample t test power calculation  
##  
##           n = 160.443  
##         delta = 0.3  
##           sd = 1  
##    sig.level = 0.05  
##         power = 0.85  
## alternative = one.sided  
##  
## NOTE: n is number in *each* group
```

Power analysis

- ▶ Two sample: define diff-in-means (δ) and SD.
- ▶ For small effects \rightarrow more 'demanding' sample

```
# Minimum sample size  
power.t.test(delta = 0.03, sd = 1, type = "two.sample",  
             alternative = "one.sided", power = 0.85)
```

```
##  
##      Two-sample t test power calculation  
##  
##              n = 15976.9  
##            delta = 0.03  
##              sd = 1  
##    sig.level = 0.05  
##            power = 0.85  
##    alternative = one.sided  
##  
## NOTE: n is number in *each* group
```

Wrapping up Week 11

- ▶ Summary:

- ▶ Testing uncertainty: beyond measures of spread.
- ▶ Hypothesis test: Proof by contradiction.
- ▶ The Null and Alternative hypotheses.
- ▶ p-value, one-sided or two-sided test.
- ▶ When to reject the null? (obtain it?)
- ▶ Type I & II errors.
- ▶ t-test for sample mean or two groups.
- ▶ Power analysis.
- ▶ Creating visual display of data - tips, software and resources.